Fuzzy Chance Constrained Optimization Approach for an Automotive Industry Model

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Abstract Optimization is an essential part of any automotive industrial management system. Supply chain management procurement and marketing activities needs a lot of handling due to its the massive number of external and internal resource constraints. The optimization seeks to a compromise points between what will be paid as managerial expenditures and what will be gained as reduction in resource prices. This paper presents a new optimization model that incorporates the uncertainties in market and negotiable parameters performed by procurement sector. Two alternative objective representation as a linear and a fractional programming functions are introduced. Parameters uncertainties were expressed using fuzzy and/or chance constraints. Market competitive prices are represented here as chance constraints giving more probability for firms to increment its prices to most permissible limits under uncertainty facing undeterministic market prices. The generated problems from the three alternative were solved and compared. The proposed models are applied to a case study of an automotive manufacturing system, where the fuzzy chance formulation has been transformed to a deterministic equivalent with a nonlinear objective function. The obtained solutions are useful for identifying sustainable management supply with maximized system profit. The results indicate that the new model representation gave better solutions concerning system profit and system cost.

Keywords: Stochastic; Chance-Constrained Programming; Fuzzy programming; Fractional Programming; Supply Chains; An Automotive Manufacturing System.

1. Introduction

Optimization theory has many applications in a wide variety of fields, e.g., the study of supply chain, finance, production, transportation, physical, and engineering design, to name a few. The optimization problems can be classified into two main approaches: deterministic optimization problems and optimization under uncertainty. Most of deterministic optimization problems in nature ignore the variance involved in the parameters. However, many of these models have parameters that are changing with time, e.g., fuel price appearing in the cost function of the model etc., and the model is optimized using a fixed value of that parameter. The results of these problems when implemented in real time, are unrealistic and unrepresentative, because the dynamic parameters are no longer at the values for which the optimization problem was solved and the results were obtained. Hence, these uncertainties can be controlled by over designing the equipment or overestimating the tuning parameters or replacing the uncertain parameters with their nominal values, thereby solving the deterministic optimization problem. Both of these methods lead to solutions that are suboptimal or sometimes infeasible. Consequently, the traditional deterministic programming methods may face dilemma in supporting Supply Chains SC management because of their weakness in reflecting uncertain information.

Therefore, the area of optimization under uncertainty appeared, aiming to improve the approaches or methodologies to create reliable solutions even in the presence of parametric uncertainty. [1]

However, the uncertainties lead to difficulties in developing optimization models for supporting decision making in SC management and getting the confidence of decisions. The uncertainty in real-world decision making originates from several sources, i.e., fuzziness, randomness, ambiguous. The two powerful approaches for mathematical programming under uncertainty are Stochastic Programming (SP) and Fuzzy Programming (FP), where the optimization problem contain some uncertain data or linguistic information. The Fuzzy Programming FP method, where the uncertainty is considered as a fuzzy set, is effective in reflecting ambiguity and vagueness in resource availabilities.

Stochastic Programming SP handles uncertain problems whose parameters' probability distributions are known. The two very popular approaches of Stochastic programming namely, Recourse programming proposed by Dantzig [2] and Chance-constrained programming proposed by Charnes and Cooper [3]. A chance-constrained programming CCP is the most prominent method used for solving optimization problems involving constraints having finite probability of being violated. Also, it shows very good characteristics in satisfying the constraints with a probability.

Several integrated fuzzy and chance-constrained programming models have been developed and applied successfully in various fields. The chance constrained programming with fuzzy parameters model was developed by [4] through predefined confidence levels of fuzzy-constraints satisfaction into optimization models. The fuzzy constraints are converted to deterministic ones at predetermined confidence levels. While [5] developed an inexact double-sided fuzzy chance-constrained programming (IDFCCP) model for agricultural water quality management under uncertainties. Similarly, [6] proposed an inexact double sided fuzzy-random-chance-constrained programming model for handling Nanshan district's air quality management problem under uncertainty.

A stochastic linear fractional programming (SLFP) approach was developed by [7] for identifying sustainable MSW management strategies under uncertainty. The SLFP method can solve ratio optimization problems associated with random information, where chance-constrained programming (CCP) was integrated into a linear fractional programming (LFP) framework. Also, [8] developed a fuzzy fractional chance-constrained programming model (FFCCPM) for supporting regional air quality management under uncertainty. [9] developed a fuzzy chance-constrained linear fractional programming method for agricultural water resources management under multiple uncertainties.

On the other hand, reducing cost or increasing profit with a capacity to deal with parameters uncertainty, whether fuzzy or stochastic, are considered important issues in an efficient supply chain. Environmental coefficients are often fuzzy or stochastic for most problems of supply chain because of incomplete information. The main aim of any supply chain planning is effective coordination and integration of the key business activities undertaken by an enterprise, starting from the

procurement of raw materials to the final products distribution to the customers.
Previously, a number of optimization models were developed representing SC model for an automotive industry [10]. The first model is crisp LP where in the second a fuzzified objective function, constraints and coefficients, developed a fuzzy LP model to analyze and predict the behavior of the system. Finally, they developed a fuzzy expert system (FES) based on rule bases that described the behavior of SC as we have extracted from and negotiated with the experts.
Fuzzy sets and rough sets were used by [11] for representing uncertainty in an automotive industry model. Although the uncertainty is desired in the previous work via fuzzy programming and rough set theory, but in the reality, the supply chain not only based on the fuzziness but also probability of achieving the constraints.
Hence, the major aims of this study entails (1) Formulation of an FCCFP model based the integration of fuzzy programming (FP), chance-constrained programming (CCP), and linear fractional programming (LFP) models. It considers the ratio between the system benefit and the system cost concurrently and reflects the system efficiency as well as handling uncertainty expressed as probability distributions or fuzziness. (2) Scenario analysis of the model applicability under multiple situations to be satisfied under specific confidence levels and reliability scenarios. (3) Application of the developed model for solving automotive industry problem and the obtained solutions are compared with some references.
The remainder of this paper is divided into the following sections: Formulation of the proposed models is presented in Section 2. In section 3, case study analysis as well as its results yielding uses of the proposed models are introduced, finally the paper is concluded in Section 4.

2. Methodology

This section introduces the detailed procedure for formulating and solving the proposed models. Before going through these algorithm steps in sections (2-2) and (2-3), the mathematical Fractional Programming Problem formulation used in representing the objective function is introduced in section (2.1) are described as follows.

2.1. Fractional Programming FP

The following general Fractional Programming Problem (FPP) model mathematical is used to representing automotive industry problem. [12]:

$$min/\,max \quad f(X) = \frac{AX + a}{BX + b} \tag{1.1}$$

$$Subject\ to$$

$$CX \le D \tag{1.2}$$

$$X \ge 0 \tag{1.3}$$

$$BX + b \ne 0 \tag{1.4}$$

where C is an auxiliary variable; X and D are column vectors, A and B are row vectors; and a & b are constants.
Fractional programming of the form (1.1) arises in a natural way such as the ratios (profit/cost), (profit/time), (cost/time), (output/input), (waste of raw material/quantity of used raw material), are to be maximized or minimized. These problems are often linear or at least concave-convex fractional programming.
Fractional programming (FP) is an effective tool for handling the optimization problems with ratio. the main advantages of FP are the flexibility, balancing conflicting objectives, also, it can compare objectives of different aspects directly through their original magnitudes and provide a balanced measure of system efficiency. It was indicated that FP could better fit the real problems through considering optimization of ratio between the physical and/or economic quantities [13].
On the other hand, there are many different direct algorithms to solve the fractional programming problem [12]. The Charnes–Cooper's transformation algorithm is considered in this paper, where the FPP is converted into an equivalent linear programming problem and solves it using already existing standard algorithms for LPP. [14].

$$max/\,min \quad \lambda \tag{1.5}$$

$$Subject\ to$$

$$C^T Y + B^T Y \ge A^T \tag{1.6}$$

$$-D^T Y + b\lambda = a \tag{1.7}$$

$$Y \ge 0 \tag{1.8}$$

where Y is a column vector; λ is a scalar; and T over the matrix denotes the transpose of the matrix.

2.2. Chance-Constrained Fractional Programming CCFP

Chance constrained programming is an effective and useful approach for treating constraints where some of the coefficients are uncertain in decision making. The decision makers are interested in satisfying an uncertainty constraint, by at least a pre-specified probability at the smallest cost.
The chance constrained fractional programming is a special type of chance constrained programming problem, where the objective function of CCFP is the ratio of two functions. The CCFP model can be expressed as follows:

$$min/\,max \quad f(X) = \frac{AX + a}{BX + b} \qquad (2.1)$$

$$Subject\ to$$

$$Pr[CX \leq D] \geq 1 - p \qquad (2.2)$$

$$MX \leq F \qquad (2.3)$$

$$X \geq 0 \qquad (2.4)$$

$$BX + b \neq 0 \qquad (2.5)$$

Where $Pr[\ \ ]$ denotes the probability of the event [ ]; p is the specified probability level of the constraint functions or confidence level of the constraints. C is a vector of coefficients in constraints; D is a random right-hand-side parameter in constraints.

The detailed solution process for CCFP can be summarized as follows:

Step 1: Formulate the original CCFP model as in (2-1) -(2-5).

Step 2: Transform the chance constraint (2.2) into its crisp equivalence as [15]:

$$\mathrm{CX} \leq \mathrm{E(D)} + \mathrm{K_p}\sqrt{\mathrm{var(D)}} \qquad (2.6)$$

For the given probability level 'p', where K_p is the standard normal value such that $\Phi(K_p) = 1 - p$ and $\Phi(K_p)$ represents the "cumulative distribution function" of the standard normal distribution evaluated at K_p.

Step 3: Formulate the deterministic LFP generated from step (2).

Step 4: Transform the FP model into a dual programming model (1.5) then solve it using LINGO Software and obtain solutions.

Step 5: Repeat steps 2–4 under different p_i levels.

Step 6: Obtaining the values for the optimum solution.

Step 7: Stop.

2.3. Fuzzy Chance-Constrained Fractional Programming FCCFP

Fuzzy Chance Constrained Fractional Programming (FCCFP) technique is suitable in handling special variable containing information which is both fuzzily imprecise and probabilistically uncertain, where the uncertain parameters is represented as fuzzy sets and probability density functions.

Generally, FCCFP model can be formulated as follows:

$$min/\,max \quad f(X) = \frac{\tilde{A}X + \tilde{a}}{\tilde{B}X + \tilde{b}} \qquad (3.1)$$

$$Subject\ to$$

$$Pos[\tilde{C}X \leq \tilde{D}] \geq \alpha \qquad (3.2)$$

$$\tilde{M}X \leq \tilde{F} \qquad (3.3)$$

$$X \geq 0 \qquad (3.4)$$

$$\tilde{B}X + \tilde{b} \neq 0 \qquad (3.5)$$

Where $Pos[\ \ ]$ denotes possibility in [ ], $\tilde{A}, \tilde{a}, \tilde{B}, \tilde{b}, \tilde{C}, \tilde{D}, \tilde{M}, \tilde{F}$ are fuzzy numbers which are expressed as fuzzy sets with membership functions $\mu(\tilde{A}), \mu(\tilde{a}), \mu(\tilde{B}), \mu(\tilde{b}), \mu(\tilde{C}), \mu(\tilde{D}), \mu(\tilde{M}), \mu(\tilde{F})$ with fuzzy tolerance of α. Each predetermined α-cut level consists of two reliability scenarios, with α-cut level under minimum and maximum reliability degrees, respectively. Generally, the problem can be expressed as:

$$Pos\{\tilde{C} \lesseqqgtr^{min} \tilde{D}\} = sup\{min(\mu_{\tilde{C}}(\tilde{x}), \mu_{\tilde{D}}(\tilde{y}))|x, y \in R, x \leq y\} \qquad (3.6)$$

$$Pos\{\tilde{C} \lesseqqgtr^{max} \tilde{D}\} = inf\{max(1 - \mu_{\tilde{C}}(\tilde{x}), 1 - \mu_{\tilde{D}}(\tilde{y}))|x, y \in R, x \leq y\} \qquad (3.7)$$

Where $\tilde{C} \lesseqqgtr^{min} \tilde{D}$ presents that the equation $\tilde{C} \leq \tilde{D}$ should be satisfied at the minimum reliability degree; $\tilde{C} \lesseqqgtr^{max} \tilde{D}$ means that the equation $\tilde{C} \leq \tilde{D}$ should be satisfied at the maximum reliability degree. Eq. (3.6) means that the possibility of $\tilde{C} \leq \tilde{D}$ is the possibility that there exists at least one pair of values $x, y \in R$ such that $x \leq y$ and the specified values of $\tilde{C}$ and $\tilde{D}$ are , x and y respectively [14].

Conversely, the mathematical concept of Eq. (3.7) is contrary to that of the Eq. (3.6).
Based on Eqs. (3.6) and (3.7), for any given $\alpha \in (0,1)$ the following two equations can be derived:

$$Pos\{\tilde{C} \lesseqqgtr^{min} \tilde{D}\} = sup\{min(\mu_{\tilde{C}}(\tilde{x}), \mu_{\tilde{D}}(\tilde{y}))|x, y \in R, x \leq y\} \geq \alpha \Leftrightarrow C^L(\alpha) \leq B^R(\alpha) \quad (3.8)$$

$$Pos\{\tilde{C} \lesseqqgtr^{max} \tilde{D}\} = inf\{max(1-\mu_{\tilde{C}}(\tilde{x}), 1-\mu_{\tilde{D}}(\tilde{y}))|x, y \in R, x \leq y\} > \alpha \Leftrightarrow C^R(1-\alpha) \leq B^L(1-\alpha) \quad (3.9)$$

where item $C^L(\alpha)$ is defined as the minimum values of all potential values at α-cut level, i.e. $C^L(\alpha) = inf\{C|C = \mu^{-1}(\alpha)\}$. Similarly, $D^R(\alpha)$ is defined as the maximum values of all potential values at an α-cut level, i.e. $D^R(\alpha) = inf\{C|C = \mu^{-1}(\alpha)\}$. μ^{-1} is the inverse function of μ.

Model (3.1) (FCCFP) can be converted into two equivalent crisp sub-models based on Eqs. (3.8) and (3.9). which correspond to the lower and upper bounds of desired objective function value under various α-cut levels.

The first sub-model (I) based on the minimum reliability is as follows:

$$min/max \quad f(X) = \frac{\tilde{A}X + \tilde{a}}{\tilde{B}X + \tilde{b}} \quad (3.10)$$

$$Subject\ to$$

$$C^L(\alpha)\, X \leq D^R(\alpha) \quad (3.11)$$

$$\tilde{M}X \leq \tilde{F} \quad (3.12)$$

$$X \geq 0 \quad (3.13)$$

$$\tilde{B}X + \tilde{b} \neq 0 \quad (3.14)$$

Whereas the second sub-model (II) based on the maximum reliability is as follows:

$$min/max \quad f(X) = \frac{\tilde{A}X + \tilde{a}}{\tilde{B}X + \tilde{b}} \quad (3.15)$$

$$Subject\ to$$

$$C^R(1-\alpha)\, X \leq B^L(1-\alpha) \quad (3.16)$$

$$\tilde{M}X \leq \tilde{F} \quad (3.17)$$

$$X \geq 0 \quad (3.18)$$

$$\tilde{B}X + \tilde{b} \neq 0 \quad (3.19)$$

The two groups of objective value and decision variables under different confidence levels α^h and constraint violation levels q_i are obtained by solving the above sub-models. The solutions can be expressed as $f^{\pm}_{opt} = [f^{-}_{opt}, f^{+}_{opt}]$ and $x^{\pm}_{opt} = [x^{-}_{opt}, x^{+}_{opt}]$ respectively.

The detailed procedures for formulating and solving the FCCFP are summarized as follows.

Step 1: Identify the uncertain variables and acquire the related fuzzy possibility distribution information.

Step 2: Construct FCCFP mathematical optimization model;

Step 3: Set an initial significance level α-cut.

Step 4: Convert fuzzy chance constraints to their respective crisp equivalents under the minimum and maximum reliability scenarios and obtain two FP models, based on FCCFP algorithm;

Step 5: Solve the two sub-models and obtained solutions;

Step 6: Repeat Steps 3–5 under different α-cut levels.

Step7: Stop.

3. Application with Results and Discussion

The following case study is given to show the application of the proposed models and algorithms.

3.1. case study overview

Supply Chains SC systems of an automotive industry consist of 6 components, as shown in Fig (1). The tasks of each enterprise, as shown in Fig. (2), are: customer, marketing and sales, manufacturing and assembly (production plant), purchasing (procurement enterprise), and parts and raw materials suppliers, where all of them are interconnected [10]. The role of marketing and procurement in selling and purchasing of goods in the SC are the main concentration of the research, with focus on:

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• The role of procurement enterprise in purchasing most proper raw materials, which directly affect the operational costs of the production plant.
• The role of procurement enterprise in negotiations with suppliers on price, transport, quality, etc.
• The role of marketing enterprise in selling the product with the maximum possible price.

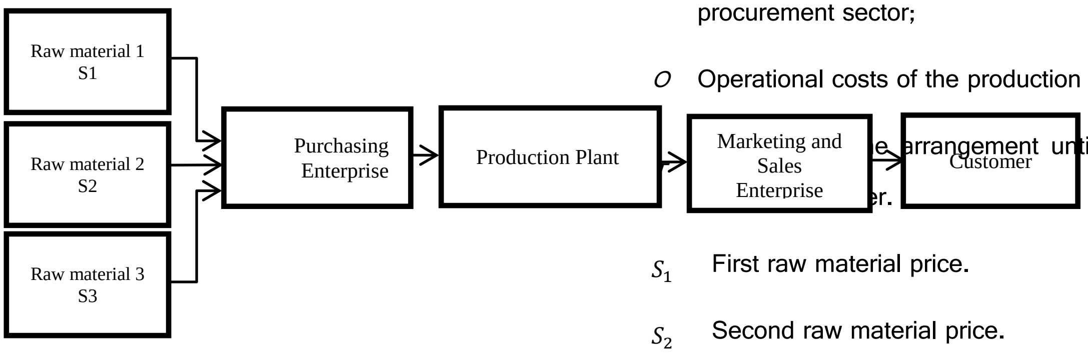

Figure 1: The component of SCM.

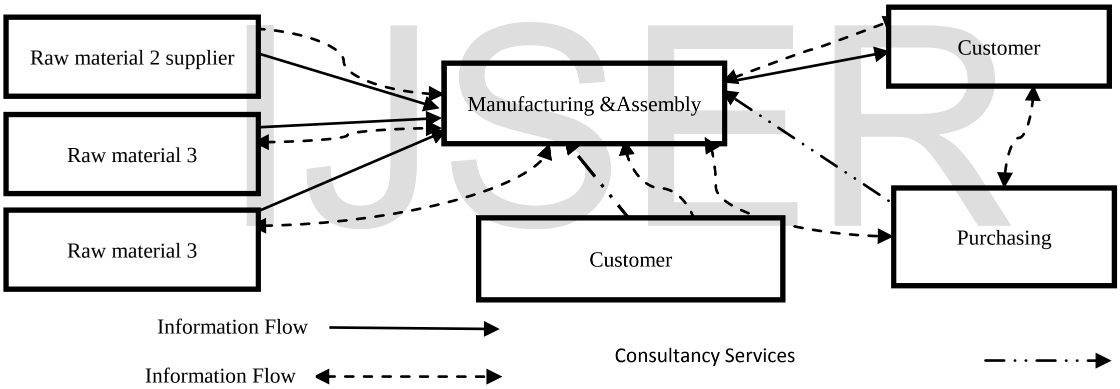

Figure 2: Relationship between different sectors of the SC

The goal of modeling the SC is maximization of profit in dollars. The variables and parameters of the system are as follows:

Variables:

P The final product price.

M Costs to be paid by production plant in marketing sector;

S Costs to be paid by production plant in procurement sector;

O Operational costs of the production plant.

e arrangement until a contract is

er.

S_1 First raw material price.

S_2 Second raw material price.

S_3 Third raw material price.

Parameters:

π_{min} Minimum price is accepted by production plant to sell its product.

π_{max} Maximum price that this product would be sold. Market situation dictate this rate.

ρ_{1min} The minimum price for first raw material.

• The role of procurement and marketing enterprises in finalizing a deal in minimum time. The stronger actions of these two enterprise will lead to sails contract for production plant in a shorter time and accordingly the investment costs of the production plant will be reduced.

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ρ_{1ma} The maximum price for first raw material.

ρ_{2mi} The minimum price for second raw material.

ρ_{2ma} The maximum price for second raw material.

ρ_{3mi} The minimum price for third raw material.

ρ_{3ma} The maximum price for third raw material.

ξ_{med} Average operational costs of the factory.

ξ_{min} Minimum operational costs of the factory.

The rate of annual interest.

I Investing for the factory production. on making moulds, prototyping, and providing other conditions;

θ_1 The rate, on which, each dollar paid in procurement enterprise will lead to a decrease in ρ_{1max} in that rate.

θ_2 The rate, on which, each dollar paid in procurement enterprise will lead to a decrease in ρ_{2max} in that rate.

θ_3 The rate, on which, each dollar paid in procurement enterprise will lead to a decrease in ρ_{3max} in that rate.

β The rate, on which, each dollar paid in marketing enterprise will lead to an increase in π_{min} in that rate.

χ The rate, on which, each dollar paid in procurement enterprise will lead to a decrease in ξ_{med} in that rate.

T_{max} Maximum acceptable time to production plant for finalizing the deal.

δ The rate, on which, each dollar paid in marketing enterprise will lead to a decrease in investment costs αIT_{max} in that rate.

ε The rate, on which, each dollar paid in a procurement enterprise will lead to a decrease in investment costs αIT_{max}.

ϕ Available budget for costing in marketing and purchasing units.

This paper aims to investigate the best way to formulate automotive industrial model as an optimization problem considering a set of uncertainties scoped model parameters.

This will include setting two different alternative representations to the objective function, one as fractional, and the other as a conventional linear representation. The interaction between the optimization model and the dynamic uncertain operational constrain parameters are to be modeled by two different formulations. The first as a chance constrained parameters and the second as a fuzzy chance constrained parameters. The proposed model will also be solved with crisp parameter values for verification purposes.

3.2. Crisp (FP) Model

$$Max\ f = \frac{P}{[(S_1 + S_2 + S_3) + S + M + O + 0.125T]} \tag{4.1}$$

$$Subject\ to$$

$$S_1 \geq \rho_{1max} - \theta_1 S \tag{4.2}$$

$$S_2 \geq \rho_{2max} - \theta_2 S \tag{4.3}$$

$$S_3 \geq \rho_{3max} - \theta_3 S \tag{4.4}$$

$$S_1 \geq \rho_{1min} \tag{4.5}$$

$$S_2 \geq \rho_{2min} \tag{4.6}$$

$$S_3 \geq \rho_{3min} \qquad (4.7)$$

$$P \leq \pi_{min} + \beta M \qquad (4.8)$$

$$P \leq \pi_{max} \qquad (4.9)$$

$$O \geq \xi_{med} - \chi S \qquad (4.10)$$

$$O \geq \xi_{min} \qquad (4.11)$$

$$\alpha IT \geq \alpha IT_{max} - \delta M - \varepsilon S \qquad (4.12)$$

$$M + S \leq \phi \qquad (4.13)$$

$$T \leq T_{max} \qquad (4.14)$$

$$(S_1 + S_2 + S_3) + S + M + O + 0.125T \neq 0 \qquad (4.15)$$

The above mathematical model is principle for all further common model representation in this paper. where its elements are defined as: Objective function (*f*) is maximizing the ratio between the price as a numerator and the expenses as a denominator. Constraints (4.2) - (4.9) are related to procurement enterprise, since this department affects purchasing raw materials from suppliers and sales of products through marketing and sales enterprise to customers. The proposed price of raw materials by each supplier is in interval $[\rho_{imin}, \rho_{imax}]$, where ρ_{imax} is first offer and $\rho_{imin}$is lowest possible price. In such a situation, production plant prefers to spend *$S* in procurement department to negotiate with suppliers to decrease ρ_{imax}. However, this investment in procurement department has different effect on price reduction, which is shown by parameters θ_i. Hence, constraints (4.2) - (4.7) try to illustrate S and S_i relations. The same situation exists for marketing department, which is shown by constrains (4-8) and (4-9). Constrains (4-10) and (4-11) show the effect of procurement department on operational costs, and (4-12) demonstrates their effect on speed up of supply and sales operation. Finally, constrains (4-13) is the budget bound and (4-14) illustrates the time restriction.

The parameters and the coefficients of the model have been determined after a long negotiation and discussion with the experts of the production plant and tuning them with some mathematical and statistical models as Table 1 [10]:

Table 1: Value of relative parameters used in the numerical experiments.

π_{min}	π_{max}	ρ_{1max}	ρ_{1min}	ρ_{2max}	ρ_{2min}	ρ_{3max}
11	16	6.5	5	17	1.2	1
ρ_{3min}	I	θ_1	θ_2	θ_3	β	χ
0.5	0.625	1.1	1.3	1.4	1.5	1.2
ξ_{med}	ξ_{min}	α	ε	ϕ	T_{max}	δ
1.2	0.8	0.2	0.05	1.5	1	0.05

The previous set of alternative objective function and constrain formulation will lead to the following problem matrix. A comparative study between the solution results of problems of Table 2 is introduced in this subsection. the crisp optimization problem depends on how to reach the most proper set of procurement investment that reach the optimal price by tuning the material price between two specific market limits. The rate θ_1 that considers reduction in cost to each paid dollar in procurement is predefined to a certain crisp value that is not always valued. However the problem tries to reach an optimal solution by varying the price of raw material between a minimum value ρ_{min} and reducible maximum value $(\rho_{max} - \theta_1 S)$. this shall interact in the optimization problem by bounding the selling price between a maximum dedicated market value (π_{max} and a minimum marginal profit value π_{min}). With the following variable results, the negotiation lead-time generated a loss of $\alpha IT = 0.05$.

Table 2: Solution of the crisp models

$Var.$	LP Solution (1000$)	FP Solution (1000$)
f	////	0.7990895
P	12.67308	12.67308
S_1	6.076923	6.076923
S_2	1.2	1.2
S_3	0.5	0.5
S	0.3846154	0.3846154
M	1.115385	1.115385
O	0.8	0.8
T	0.4	0.4
$Profit\ (Z)$	2.546157	2.546157

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3.3. Chance-Constrained Fractional Programming (CCFP)

$$Max\ f = \frac{P}{[(S_1 + S_2 + S_3) + S + M + O + 0.125T]} \quad (5.1)$$

$$Subject\ to$$

$$Pr[S_1 + \theta_1 S \geq \rho_{1max}^{\pm}] \geq 1 - p \quad (5.2)$$

$$Pr[S_2 + \theta_2 S \geq \rho_{2max}^{\pm}] \geq 1 - p \quad (5.3)$$

$$Pr[S_3 + \theta_3 S \geq \rho_{3max}^{\pm}] \geq 1 - p \quad (5.4)$$

$$Pr[S_1 \geq \rho_{1min}^{\pm}] \geq 1 - p \quad (5.5)$$

$$Pr[S_2 \geq \rho_{2min}^{\pm}] \geq 1 - p \quad (5.6)$$

$$Pr[S_3 \geq \rho_{3min}^{\pm}] \geq 1 - p \quad (5.7)$$

$$Pr[P - \beta M \leq \pi_{min}^{\pm}] \geq 1 - p \quad (5.8)$$

$$Pr[P \leq \pi_{max}^{\pm}] \geq 1 - p \quad (5.9)$$

$$O \geq \xi_{med} - \chi S \quad (5.10)$$

$$O \geq \xi_{min} \quad (5.11)$$

$$\alpha IT \geq \alpha IT_{max} - \delta M - \varepsilon S \quad (5.12)$$

$$M + S \leq \phi \quad (5.13)$$

$$T \leq T_{max} \quad (5.14)$$

$$(S_1 + S_2 + S_3) + S + M + O + 0.125T \neq 0 \quad (5.15)$$

Chance constrained optimization problem, the constrains check for the most proper point that all parameters different event probability space meet at, i.e. the probability each cost parameters will meet the floor ρ_{max}^{-} or ceiling ρ_{max}^{+} after adding the penalty while is the procurement change for any cost reduction. The same goes for the selling price between $\pi_{max}^{\pm}$. The solution was generated using different values of confidence giving the following optimal results.

Fig. 3 show the distribution information of uncertain parameters under different probability levels of constraint violation (p_i). For example, if ρ_{1max} is 5.716678 under $p_i = 0.003$, which indicates that such a ρ_{1max} can be guaranteed with a probability of $1 - p_i = 0.999$. Table 3 lists the distribution information of material and product prices under different probability levels of constraint violation (p_i).

Table 3: Some parameters under different pi levels.

p_i level	ρ_{1max}	ρ_{2max}	ρ_{3max}	ρ_{1min}	ρ_{2min}	ρ_{3min}	π_{min}	π_{max}
0.0003	5.716678	1.543356	0.843336	4.716678	0.808356	0.265	11.21667	15.71668
0.001	5.741677	1.548354	0.848335	4.741677	0.820854	0.2725	11.24167	15.74168
0.05	5.863339	1.572678	0.872668	4.8633388	0.8816776	0.309	11.36333	15.86334
0.1	5.893338	1.578675	0.878668	4.8933376	0.8966752	0.318	11.39333	15.89334
0.25	5.944169	1.588838	0.888834	4.9441689	0.9220878	0.33325	11.44417	15.94417
0.5	6	1.6	0.9	5	0.95	0.35	11.5	16
0.75	6.056664	1.611329	0.911333	5.0566644	0.9783288	0.367	11.55667	16.05666
0.9	6.107496	1.621491	0.921499	5.1074957	1.0037414	0.38225	11.6075	16.1075
0.95	6.137495	1.627489	0.927499	5.1374945	1.018739	0.39125	11.6375	16.13749
0.99	6.194159	1.638818	0.938832	5.1941589	1.0470678	0.40825	11.69417	16.19416
0.9998	6.290822	1.658143	0.958164	5.2908217	1.0953934	0.43725	11.79083	16.29082

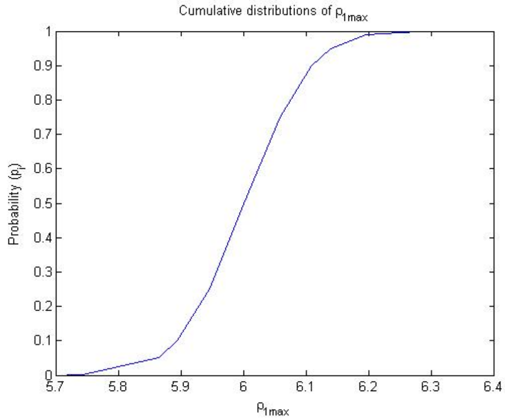
Table 4: Results of the CCFP model under different pi levels.

p_i level	0.0003	0.001	0.05	0.1	0.25	0.5	0.75	0.9	0.95	0.99	0.999
f	1.486853	1.481733	1.4576	1.451841	1.442251	1.43195	1.421737	1.412776	1.407574	1.397918	1.381942
P	12.10303	12.12803	12.24969	12.27969	12.33053	12.38636	12.44303	12.49386	12.52386	12.58053	12.67719
S_1	4.716678	4.741677	4.863339	4.893338	4.944169	5	5.056664	5.107496	5.137495	5.194159	5.290822
S_2	0.808356	0.820854	0.881678	0.896675	0.922088	0.95	0.978329	1.003741	1.018739	1.047068	1.095393
S_3	0.265	0.2725	0.309	0.318	0.33325	0.35	0.367	0.38225	0.39125	0.40825	0.43725
S	0.909091	0.909091	0.909091	0.909091	0.909091	0.909091	0.909091	0.909091	0.909091	0.909091	0.909091
M	0.590909	0.590909	0.590909	0.590909	0.590909	0.590909	0.590909	0.590909	0.590909	0.590909	0.590909
O	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
T	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
Profit (Z)	**3.963**	3.943	3.84567	3.82168	3.78102	3.73636	3.69104	3.65037	3.62638	3.69421	3.50373

Table 5: Results of the CCLP model under different pi levels.

p_i level	0.0003	0.001	0.05	0.1	0.25	0.5	0.75	0.9	0.95	0.99	0.999
P	12.61859	12.65225	12.81602	12.85641	12.92484	13	13.07629	13.14471	13.1851	13.26138	13.3915
S_1	5.094755	5.1261	5.278647	5.316261	5.379996	5.45	5.521049	5.584784	5.622399	5.693447	5.814649
S_2	0.808356	0.820854	0.881678	0.896675	0.922088	0.95	0.978329	1.003741	1.018739	1.047068	1.095393
S_3	0.265	0.2725	0.309	0.318	0.33325	0.35	0.367	0.38225	0.39125	0.40825	0.43725
S	0.565385	0.559615	0.531539	0.524615	0.512885	0.5	0.486923	0.475192	0.468269	0.455192	0.432885
M	0.934615	0.940385	0.968462	0.975385	0.987115	1	1.013077	1.024808	1.031731	1.044808	1.067115
O	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
T	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
Profit (Z)	**4.10048**	4.0828	3.996698	3.975471	3.939509	3.9	3.859908	3.823936	3.802708	3.762616	3.694211

Table 4 shows the results of CCFP model under different probability levels of constraint violation (p_i). The price and cost variables generated values are listed at each (p_i) level as well as the profit generated at that level. The same goes for Table 5 that shows the same result under different probability levels (p_i) but this time for the CCLP model. From the two tables it is obvious that the results obtained using CCLP is better giving a higher profit at the same (p_i).



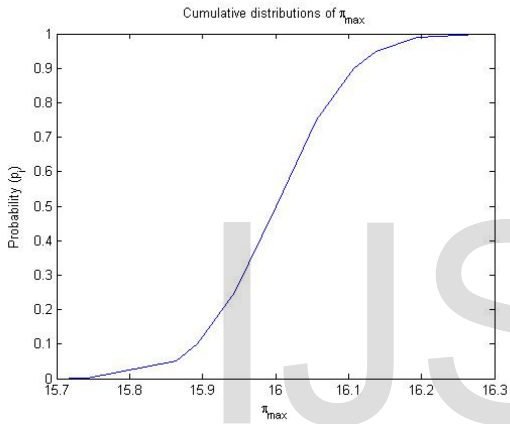


Figure 3: Cumulative distributions of some parameters

3.4. *Fuzzy Chance-Constrained Fractional Programming (FCCFP)*

$$Max\ f = \frac{P}{[(S_1 + S_2 + S_3) + S + M + O + 0.125T]} \quad (6.1)$$

$$Subject\ to$$

$$Pr[S_1 + \tilde{\theta}_1 S \geq \rho^{\pm}_{1max}] \geq 1 - p \quad (6.2)$$

$$Pr[S_2 + \tilde{\theta}_2 S \geq \rho^{\pm}_{2max}] \geq 1 - p \quad (6.3)$$

$$Pr[S_3 + \tilde{\theta}_3 S \geq \rho^{\pm}_{3max}] \geq 1 - p \quad (6.4)$$

$$Pr[S_1 \geq \rho^{\pm}_{1min}] \geq 1 - p \quad (6.5)$$

$$Pr[S_2 \geq \rho^{\pm}_{2min}] \geq 1 - p \quad (6.6)$$

$$Pr[S_3 \geq \rho^{\pm}_{3min}] \geq 1 - p \quad (6.7)$$

$$Pr[P - \tilde{\beta} M \leq \pi^{\pm}_{min}] \geq 1 - p \quad (6.8)$$

$$Pr[P \leq \pi^{\pm}_{max}] \geq 1 - p \quad (6.9)$$

$$O \geq \xi_{med} - \tilde{\chi} S \quad (6.10)$$

$$O \geq \xi_{min} \quad (6.11)$$

$$\alpha IT \geq \alpha IT_{max} - \tilde{\delta} M - \tilde{\varepsilon} S \quad (6.12)$$

$$M + S \leq \phi \quad (6.13)$$

$$T \leq T_{max} \quad (6.14)$$

$$(S_1 + S_2 + S_3) + S + M + O + 0.125T \neq 0 \quad (6.15)$$

Considering that the procurement effort for price reduction are not certain (e.g. $\theta, \beta, \chi, \delta, \varepsilon$) the previous chance constrained model can be modified to a fuzzy chance constrained model so as to incorporate the uncertainty these parameters. This would add new steps to the solution algorithm. The solution algorithm tries to adjust the probability of reaching a certain raw material price, delay and cost values, while a parallel variation occurs to the rates paid to the procurement sector based on how far did they reached the target price value. The fuzzy variable is estimated between left and right confidence value that depends on supremum and infimum of the set of each parameter vailed membership function.
Table 6 and Table 7 show the results of FCCFP model under different α cut levels left and right oriented respectively. while Table 8 and Table 9 show the results of FCCLP model under different α cut levels left and right oriented respectively.

Table 6: FCCFP under α^l *level*

α^l level	0.2	0.4	0.6	0.8
f	1.408901	1.393126	1.411835	1.421013
P	13.37864	13.70849	13.62955	13.42857
S_1	5.731068	5.922642	5.852893	5.707143
S_2	1	1.1	1.05	1

S_3	0.38	0.44	0.41	0.38
S	0.38835	0.37735	0.33057	0.357143
M	1.11165	1.1226	1.1694	1.142857
O	0.8	0.8	0.8	0.8
T	0.677864	0.6196	0.32711	0.502857
$Profit\ (Z)$	3.88284	3.86845	3.9758	**3.97857**

Table 7: FCCFP under $\alpha^R\ level$

$\alpha^R\ level$	0.2	0.4	0.6	0.8
f	1.458824	1.504529	1.549274	1.536884
P	12.4	12.47115	13.50455	11.1
S_1	4.9	4.8	5.463636	4.6
S_2	0.9	0.85	0.9	0.75
S_3	0.32	0.29	0.32	0.23
M	0.8333	0.6923	0.363636	0.74074
S	0.666	0.80769	1.136364	0
O	0.8	0.8	0.448436	0.8
T	0.64	0.3926	0.677	0.8133
$Profit\ (Z)$	3.9007	4.1821	**4.787852**	3.8776

Table 8: FCCLP under $\alpha^l\ level$

$\alpha^l\ level$	0.2	0.4	0.6	0.8
P	13.37864	13.7084	13.62955	13.42857
		9		
S_1	5.731068	5.922642	5.852893	5.707143
S_2	1	1.1	1.05	1
S_3	0.38	0.44	0.41	0.38
S	0.38835	0.37735	0.33057	0.357143
M	1.11165	1.1226	1.1694	1.142857
O	0.8	0.8	0.8	0.8
T	0.677864	0.6196	0.32711	0.502857
$Profit\ (Z)$	3.88284	3.86845	3.9758	**3.978571**

Table 9: FCCLP under $\alpha^R\ level$

$\alpha^R\ level$	0.2	0.4	0.6	0.8
P	13.10455	12.93029	14.57864	12.5864
S_1	5.463636	5.330423	5.731068	5.061399
S_2	0.9	0.85	1	0.75
S_3	0.32	0.29	0.38	0.23
S	0.36363	0.37566	0.38835	0.398964
M	1.13636	1.12433	1.11165	1.101036
O	0.8	0.8	0.8	0.8
T	0.504727	0.563683	0.677864	0.238839
$Profit\ (Z)$	4.05783	4.08942	**5.08284**	4.215145

Table 10: The comparison between the obtained result by the prosed models and the some references

	Model Type	Solution Profit (Z)
[10]	LP crisp model	2.54
	LP model with fuzzy coefficients	2.64
	LP model with fuzzy constraints and objective function	3.10
	LP model with fuzzy coefficient, constraints, and objective functions	3.154

	Fuzzy rule base model, with inputs from last LP model		4.78
[11]	Crisp		2.5346
	fuzzy	Trapezoidal membership (Trapmf)	2.58
		Triangle membership (trimf)	2.6147
		Fuzzy constraints and objective function	2.9226
	Rough		[2.5346, 3.0467]
The proposed models	Fractional Programming FP		2.546157
	Linear Programming LP		2.546157
	Fuzzy Chance Constrained Linear Programming CCLP		4.10048
	Chance Constrained Fractional Programming CCFP		3.963
	Fuzzy Chance Constrained Fractional Programming FCCFP		4.787852
	Fuzzy Chance Constrained Linear Programming FCCLP		5.08284

Table 10 shows a comparison between the proposed models and two other references results for the same problem. For the Automotive Industry model of this paper, the best previously obtained result was that of [10] which could be indicated from table(10). In this model a fuzzy rule-based expert system was composed which is a problem specific tedious operation. However, the FCCLP managed to get a better optimization value of 5.08284. It could also be concluding from the table results that LP is slightly better than FP in representing the present problem. It also could be clearly noticed that combining fuzzy parameter representation to chance constrained optimization is recommended for better optimization results.

4. Conclusions

For automotive industry application model, two different LP and FP models were introduced as optimization models. The models were solved as crisp models. The uncertainty in the model parameters was represented using chance constrained and fuzzy chance constrained. Six different models were generated from the proposed modeling assumption. A comparative study between six model results and the previous obtained results was carried out. The comparison proved the superiority of fuzzy chance constrained optimization based on LP representation among all other algorithms. Further, modelling considerations incorporating game theory negotiation techniques could be investigated in future work.

References

[1] N. Virivinti and K. Mitra, “Intuitionistic Fuzzy Chance Constrained Programming for Handling Parametric Uncertainty : An Industrial Grinding Case Study,” 2015.

[2] G. Dantzig, “Linear programming under uncertainty,” *Manage. Sci.*, 1955.

[3] A. Charnes and W. Cooper, “Chance-constrained programming,” *Manage. Sci.*, 1959.

[4] B. Liu and K. Iwamura, “Chance constrained programming with fuzzy parameters,” *Fuzzy Sets Syst.*, vol. 94, no. 2, pp. 227–237, 1998.

[5] Y. Xu and X. S. Qin, “Agricultural effluent control under uncertainty: An inexact double-sided fuzzy chance-constrained model,” *Adv. Water Resour.*, vol. 33, no. 9, pp. 997–1014, 2010.

[6] L. Shao, Y. Xu, and G. Huang, “An inexact double-sided chance-constrained model for air quality management in Nanshan District, Shengzhen, China,” *Eng. Optim.*, 2014.

[7] H. Zhu and G. Huang, “SLFP: a stochastic linear fractional programming approach for sustainable waste management,” *Waste Manag.*, 2011.

[8] F. Liu, Z. Wen, and Y. Xu, “A fuzzy fractional chance-constrained programming model for air quality management under uncertainty,” *Eng. Optim.*, no. September, pp. 1–19, 2015.

[9] P. Guo, X. Chen, M. Li, and J. Li, “Fuzzy chance-constrained linear fractional programming approach for

optimal water allocation," *Stoch. Environ. Res. risk*, 2014.
[10] M. F. Zarandi and M. F. Zarani, "Five crisp and fuzzy models for supply chain of an automotive manufacturing system," *Int. J.*, 2007.
[11] D. Gomaa, O. Abdel-Raouf, and H. Abdel-Kader, "Handling uncertainty in supply chain management," in *The 5th International Conference on ICT in our lives 2015*, 2015.
[12] O. A. Raouf and I. M. Hezam, "Solving Fractional Programming Problems based on Swarm Intelligence," *J. Ind. Eng. Int.*, vol. 10, no. 2, p. 56, Jun. 2014.
[13] H. Zhu, W. W. Huang, and G. H. Huang, "Planning of regional energy systems: An inexact mixed-integer fractional programming model," *Appl. Energy*, vol. 113, pp. 500–514, 2014.
[14] L. Cui, Y. Li, and G. Huang, "Double-sided fuzzy chance-constrained linear fractional programming approach for water resources management," *Eng. Optim.*, 2016.
[15] S. Pramanik and D. Banerjee, "Chance Constrained Multi-Objective Linear Plus Linear Fractional Programming Problem Based on T aylor ' s Series Approximation," vol. 1, no. 3, pp. 55–62, 2012.